1. Course name (in Polish)	2. Revision date
Algebra liniowa i geometria	2020-09-15

3. Course name (in English)	4. Course code
Linear Algebra with Geometry	ALG

5. Unit responsible	6. ECTS score
Faculty of Information Technology /Chair of Algorithms, Intelligent Systems and Mathematics.	5

7. Studies

Field of Study	Level	Study mode	Specialization	Semester
Computer Science	undergraduate	full-time	n/a	2

8. Instructor or supervisor

dr hab. Ewa Turska

9. Type of classes and number of hours

Number of hours		Number of units	
Lecture	**Class/ Lab**	**Lecture**	**Class/ Lab**
30	30	15	15

10. Course language

English

11. Methods of instruction

Lecture:

- Lecture including discussions and multimedia material.

Class/ Lab:

- Discussion and solutions of problems in classroom, home assignments

12. Grading policy

A.1 Type of classes	A.2 Completion
Class/ Lab	• Graded credit
Lecture	• Exam

B. Grading procedure

Class/ Lab

- Two~~/ Three four~~ tests with retakes shall be conducted semester-wise.

Lecture

- A final written examination shall be conducted at the end of the semester

13. Prerequisites and co-requisites

Course name and code	Required knowledge and skills
High School mathematics course	Algebra of real numbers, ability to solve simple linear equations, elementary properties of trigonometric functions.

14. Course goals and learning outcomes

A. **Abstract in Polish**

Celem przedmiotu jest zapoznanie studentów z podstawowymi pojęciami algebry liniowej jak i nauczenie umiejętności ich stosowania. W szczególności nauczenie umiejętności stosowania liczb zespolonych, zrozumienie metody Eliminacji Gaussa, wykonywania działań na macierzach, obliczania wyznaczników, wyznaczania macierzy odwrotnej, wyznaczania rozwiązań układów równań liniowych oraz ich interpretacji w terminach macierzy, kombinacji wektorów i transformacji liniowej, rozpoznania przestrzeni liniowej, wyznaczenia współrzędnych w różnych bazach, znajdowania macierzy przekształceń liniowych w bazach, obliczania wartości własnych i wektorów własnych przekształcenia liniowego, diagonalizacji macierzy oraz stosowania iloczynu skalarnego do wyznaczania baz ortogonalnych. W trakcie kursu zostaną przedstawione związki z informatyką, w szczególności grafiką komputerową, bazami danych, algorytmami, szacowaniem liczb działań arytmetycznych.

B. **Abstract in English**

The aim of the course is to provide the students with a good understanding of the important concepts of linear algebra and the ability to use them. That is to manage with the main functions of complex variables, matrices and determinants, perform Gaussian Elimination, find the inverse matrix, solve systems of linear equations and be able to interpret them in terms of matrices, linear combinations of vectors and linear transformations, distinguish linear spaces, find coordinates relative to different bases, use linear transformations and find their matrices relative to different bases, determine eigenvectors and eigenvalues, diagonalize a matrix, use the dot product to find orthogonal bases.

Connect Linear Algebra to other fields especially those appearing in Computer Science such as computer graphics, data bases, algorithms, running time estimates etc.

15. Course structure

Topic no	Lecture	Class/ Lab
1	**Preliminaries, Complex numbers I.** Review of the dot product**.** Arithmetic of complex numbers, conjugation, Argand diagram. Solution of quadratic equation.	Examples and practise problems to lecture 1, revision of trigonometric functions, equation of a circle.
2	**Complex numbers II.** Modulus, argument, polar form, de Moivre'a Formula	Examples and practise problems to lecture 2.
3	**Complex numbers III.** Roots of unity, roots of arbitrary degree, exponential form. ~~Riemann sphere~~	Examples and practise problems to lecture 3.
4	**Polynomials with real and complex coefficients.** Roots, divisibility, Remainder Theorem, the Fundamental Theorem of Algebra	Examples and practise problems to lecture 4.
5	**Linear systems of equations**. Echelon form of a matrix, elementary operations, Gaussian Elimination, pivot. Computational aspects.	Examples and practise problems to lecture 5
6	**Matrix algebra**. Block matrix, ~~matrix rank I~~. Elementary row operations as matrix multiplication.	Examples and practise problems to lecture 6. Revision.
7	**Determinants**. The origin, definitions, geometrical interpretation. Properties of determinants, matrix rank	Examples and practise problems to lecture 8
8	**Inverse matrix** – cofactor and row reduction methods, solving systems of equations using inverse matrix.	Examples and practise problems to lecture 9.
9	**Cramer's Rule, Kronecker-Capelli Theorem**. Homogeneous and non- homogeneous systems of equations.	Examples and practise problems to lecture 10.
10	**Linear spaces. Vectors**, spanning sets. Linear combinations. Elementary Row Operations as linear combinations.	Examples and practise problems to lecture 11.
11	**Basis of linear space**. Coordinates, change-of-basis matrix.	Examples and practise problems to lecture 12.
12	**Linear transformation**. Matrix of linear transformation. Kernel and Image. Connections with rank. Interpretation of elementary row operations ~~as linear transformations.~~	Examples and practise problems to lecture 13.
13	**Eigenvectors and eigenvalues**. Matrix diagonalization.	Examples and practise problems to lecture 14.
14	**Euclidean Spaces** Inner product. Orthogonality. Orthogonal basis.	Examples and practise problems to lecture 15. Revision.
15	**Analytical Geometry Revision**	

16. Online resources:

- Wolfram-Alpha

17. Readings

A. Recommended texts

- "Matrix Analysis and Applied Linear Algebra", C. D. Meyer, SIAM.
- "Linear Algebra and Its Applications", D.C. Lay
- "Theory and problems of matrices", F. Ayres Jr, Schaum Publishing, New York

B. Other readings

- „Algebra Liniowa 1, 2", T. Jurlewicz, Z. Skoczylas, GiS Wroclaw
- "Linear Algebra with Applications", G. Williams

Major learning outcomes	Course learning outcomes	Learning outcome verification methods (using the procedure specified in 12B)
18. Knowledge gained during the course		
I1_W01	Algebra of complex numbers	Written test
I1_W01	Roots of polynomials	Written test
I1_W01	Solutions of systems of equations	Written test
I1_W01	Algebra of matrices	Written test
I1_W01	Linear Vector Spaces	Written test
I1_W01	Linear Transformations	Written test
I1_W01	Eigenvalues and eigenvectors	Written test
I1_W01	Geometry in 3 dimensions	Written test
19. Skills gained during the course		
I1_U09	Ability to add, multiply, find rational roots and convert complex numbers from algebraic to trigonometrical or exponential form.	Written test
I1_U09	Solve complex and real equations.	Written test
I1_U09	Solve system of equations using elementary operations, determinants or inverse matrices.	Written test
I1_U09	Find sum, product of matrices, determine inverse matrix, calculate determinant.	Written test
I1_U09	Add vectors, determine linear dependence, find basis and dimension of linear space, find coordinates of vector.	Written test
I1_U09	Determine matrix of linear transformation its kernel and image.	Written test
I1_U09	Find eigenvalues and eigenvectors of linear transformation or matrix.	Written test
I1_U09	Determine orthogonality of vectors, write the equation of a line or plane in 3D.	Written test
20. Competences gained during the course		
I1_K07	Competent in algebra of complex numbers, matrices and vectors	Written test
I1_K07 I1_K08	Can associate abstract terms, and perform trains of thought	Written test

21. Facilities required

Computer workstation features	Software	Equipment

22. Certificates in the field covered by the course

- none

23. Course rationale (practical applications and related research topics)

A. Companies and industry branches requiring the knowledge and skills from the field of this course:

Engineering computer software companies specializing in Data Base, Computer Graphics, Artificial Intelligence, Machine learning, Computer vision, Search Engines etc.

B. Jobs the knowledge and skills from the field of this course are essential for:

Software engineer, Computer Engineer, Animator, Research Analyst, Special effects director, Computer Games Designer, Cryptanalyst, Software architect etc.

C. Example thesis subjects or research project topics:

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